Table 1 Effect of Q multiplication factor on IIP errors

Nominal trajectory	Multiplication factor of Q				
	1	2	4	8	16
Nominal trajectory					
Down-range rms error, km	6.9	7.1	7.8	8.6	10.2
Cross-range rms error, km	2.8	3.0	3.1	3.2	3.4
Failure mode trajectory					
Down-range rms error, km	7.2	7.3	7.7	8.2	9.0
Cross-range rms error, km	45.3	39.0	32.6	26.4	21.1

Table 2 Comparison of QDF and LKF

Filter	Impact range, km	Down-range rms error, km	Cross-range rms error, km
QDF	4088	36.0	14.0
LKF	4088	6.9	2.8

a 3000-km impact range. The IIP trace obtained for one of the SLV3 flights using both LKF and QDF is given in Fig. 2, which clearly shows the better performance of the LKF.

Additionally, an assessment of radar noise could be made by determining the deviation of the real measurements against the estimated measurements. Further, the editing of wild measurements by comparing the residuals with their variances helped to identify radar malfunction in real time. In fact, the LKF outperforms for real flight data with good model compensation and noise suppression.

Conclusions

A new approach to estimate model uncertainty by processing the ideal state has been established in this paper. The simulation results of nominal and failure mode trajectories as well as real flight data processing show the potential of this method for accurate real-time state estimation for range safety application.

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Optimal Team Tactics

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Introduction

WHEN a number of high-speed aerial threats are to be intercepted by a different number of defensive vehicles, it may not be obvious how to assign pursuers to evaders and vice versa. This assignment can be such as to optimize, in the differential-game sense, the total increase in range of the offensive evaders. The assignments should basically be such that this score is larger if any pursuer maneuvers otherwise and is smaller if any evader steers suboptimally.

Assignment implies that each evader steers so as to maximize the downrange intercept with respect to an optimal pursuer. Each pursuer is steering to minimize the downrange gain of his assigned evader. When the pairs do not interact with one another, the geometry thus implies an optimal one-to-one pairing of pursuers and evaders.

The evaders are located arbitrarily in the horizontal plane, and all motion is in this plane. The defending pursuers are located elsewhere in the plane, perhaps between the evaders and the "goal" line being defended. In the absence of sensed pursuers, the evaders would follow parallel paths toward this goal line.

The simplest version of thus guidance problem requires tactics "in the large," when maneuver transients can be ignored and velocity direction discontinuities are permitted. The amorphous character of such "team" dynamics, in which components are controlled independently, has apparently been examined only recently.[1] Most air combat studies[2-5] have dealt with one-on-one dynamic models, focusing on range optimization or time optimization for short-range geometries. In these prior versions of the combat problem, more constraints are present (turn rate limits, acceleration limits, weapon-envelope boundaries and limits, etc.) than in the present model, and usually only the *relative* geometry is of interest, since the weapon envelopes are considered to be fixed to the vehicles.

Analysis

A number n of incoming threats (hypersonic vehicles, aircraft, cruise missiles, etc.) are to be intercepted by a number m of defensive aircraft or missiles. The analytical model of this representative m-on-n problem is extremely simple, since heading transients are immediate and the speeds are constant. Because the pursuers are faster, the capture radius can be zero, and all trajectories are straight lines. The coplanar motion avoids complexities that are secondary with respect to the assignment algorithm. Data availability (e.g., from satellites) of all positions is assumed for implementation by both teams.

The one-on-one problem is shown in Fig. 1, with the evader E and the pursuer P as shown. The y axis is the direction in which E wishes to maximize the range gain, by choice of H_e. The heading of the pursuer, H_p, minimizes this distance. The speed ratio is $V_e/V_p = \gamma < 1$.

In these axes, P is at (x_p, y_p), and the intercept point of maximum ordinate is

$$x_f = -\gamma^2 x_p/(1-\gamma^2) \tag{1}$$

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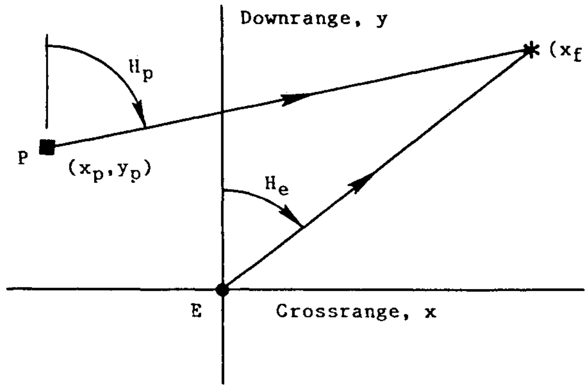

Fig. 1 Initial geometry, one-on-one encounter.

$$y_f = (-\gamma^2 y_p + \gamma r_p)/(1-\gamma^2) \tag{2}$$

where r_p is the initial range.

The optimal headings of evader and pursuer are then seen to be

$$H_e = \arctan(-x_f/y_f) \tag{3}$$

and

$$H_p = \arctan[(x_p - x_f)/(y_f - y_p)] \tag{4}$$

The intercept time must then be $t_f = \sqrt{x_f^2 + y_f^2}/V_e$. If P steers incorrectly, E's optimal heading will change with time, and the intercept point (if any) will have a larger y coordinate. Likewise, if only E plays incorrectly, the y_f coordinate will be smaller. This is the saddle point characteristic[2] of differential game tactics.

If the defensive vehicles outnumber the offensive team $(m > n)$, potential complexities occur if assignments can be of the "two-on-one" type. (Since the dynamics are planar, only two pursuers are required. But it must be determined which evaders should be assigned two pursuers, etc.) It is assumed in the examples to be shown here that only the optimal pursuer is utilized, i.e., only the specific P for which y_f is the least of the m values is of concern to E. Likewise, if there are several evaders and a single pursuer, the pursuer intercepts the evader for which y_f has the least value. The score is minimized by assigning the m best pursuers, leaving the others unassigned, as reserves.

If each evader maximizes its gain with respect to its optimally assigned pursuer, and no interference occurs with advancing time, the optimal score is the sum of n individual optimal scores,

$$\min_{i} J = \min_{i} \sum_{i=1}^{n} y_f(i,j) \tag{5}$$

where only n of the m pursuers is active. The score $y_f(i,j)$ is the least of n optimal one-on-one scores, each of which results from an optimal heading of evader j.

The dynamic equations under the assumptions given are trivial; i.e.,

$$\dot{x}_e(j) = \gamma \sin H_e(j)$$

$$\dot{y}_e(j) = \gamma \cos H_e(j) \tag{6}$$

and

$$\dot{x}_p(i) = \sin H_p(i)$$

$$\dot{y}_p(i) = \cos H_p(i) \tag{7}$$

For any geometry, whatever the numbers m and n, the distances y_f are found from Eq. (2) and entered into a matrix (Table 1).

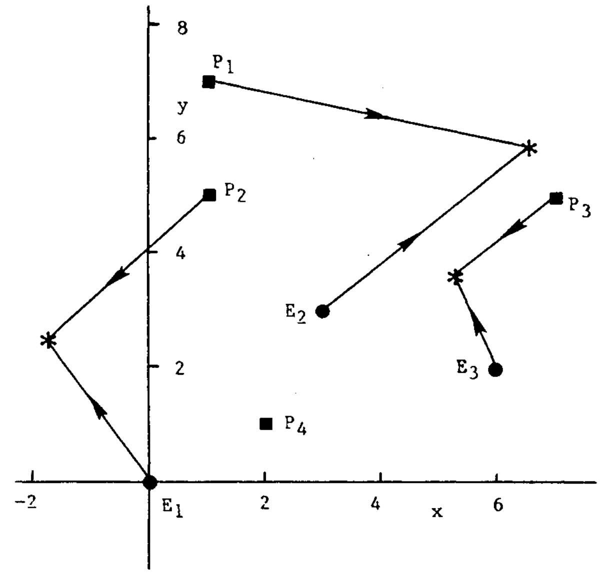

Fig. 2 Example optimal trajectories, speed ratio = 0.80.

Table 1 Downrange intercepts, $y_f(i,j)$

Pursuer number	Evader number 1	2	...	n
1	$y_f(1,1)$	$y_f(1,2)$	...	$y_f(1,n)$
2	$y_f(2,1)$	$y_f(2,2)$	...	$y_f(2,n)$
...	...	...	...	...
m	$y_f(m,1)$	$y_f(m,2)$	...	$y_f(m,n)$

Table 2 Example downrange intercepts, $y_f(i,j)$

Pursuer number	Evader number[a] 1	2	3
1	3.27	*2.83	6.82
2	*2.44	2.73	7.62
3	10.23	6.38	*1.69
4	*3.19	8.52	7.38

[a]Asterisk indicates the initial choice of evader for each pursuer.

A four-on-three example is shown in Fig. 2. The evaders are moving at Mach 8, and the pursuers at Mach 10, so $\gamma = 0.8$. Intercept ranges are given in Table 2. The initial choice of evader for each pursuer is indicated by the asterisk, under the assumption that this pursuer has no teammates, each choice being the smallest score in that row. But the pursuers must minimize the total range of the three evaders, so the optimal assignment of evaders to pursuers is

$$(P_i, E_j) = (1,2),\ (2,1),\ (3,3)$$

which corresponds to the optimal score $J = 6.96$. Other choices by the pursuers imply higher scores. Pursuer 4 is not assigned a target, despite its apparently favorable initial location, because its intercept score for each evader exceeds that of the best (minimizing) pursuer for that evader. It may happen that two or more evaders are thus paired with the same pursuer; in this case, second and third choices must be made from the table, such that the sum of Eq. (5) is minimized.

Evader 1 and pursuer 4 of Fig. 2 are taken as a typical one-on-one pairing. The optimal evader heading is found from Eqs. (1) and (2) as $H_e = 48.09$ deg, and the optimal range gain from the indicated initial geometry is $y_f = 3.19$.

Certain initial geometries have an apparent initial pairing, but if evasion of one pursuer means moving toward *another*

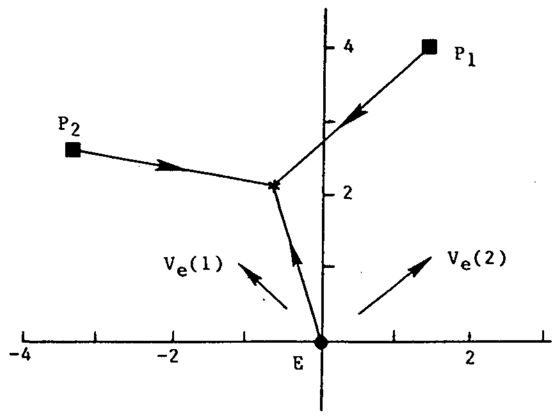

Fig. 3 Two-on-one optimization.

pursuer, additional complexities enter the problem. Figure 3 shows a two-on-one geometry. Here the circles of "reachable points" about each vehicle have an earliest common intersect time, and all three vehicles head toward that point. For higher dimensional interfering problems, the assignments and the tactics are less evident. For example, optimal evasion from one pursuer may mean that an evader moves toward a second pursuer. Such coupled trajectories remain an open question, even before real-space constraints are imposed.

Discussion

Tactics are determined in the closed-loop sense, so suboptimal play by a single pursuer or evader could change the trajectories of all of the vehicles from that time forward. This was illustrated in Ref. 1 for a six-player example from football, for which the *evaders* were faster. Therefore, frequent position updates are required if each team is to profit from tactical errors of the other. Such errors may mean that assignments can change with time.

In addition to the potential complexities caused by trajectory interference or coupling, many practical features make the assignment problem more complex than simple examples suggest. For example, a detailed dynamic model near the time of intercept may be needed to answer the question, "Is this a hit or a near miss?"

Other operational refinements of potential importance could include differing noise levels in the position data of the vehicles, modification for spherical coordinates at long range, transient time intervals for maneuvers, range and angle limits of sensors, specification of discrete targets instead of a goal line, ranking the importance of such individual targets, varying the altitudes of individual vehicles, differing and variable speeds of individual vehicles, and real-space constraints.

Conclusions

The assignment matrix formulation appears to be a practical approach to the solution of high-order versions of the team-intercept problem. If the heading and speed transients can be ignored, the total downrange gain is a performance index that can be optimized by the two teams. The guidance law is simple because the pursuers are assumed to be faster than the evaders, unless the geometry implies changes with time of the assignments.

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Design of Low-Sensitivity Modalized Observers Using Left Eigenstructure Assignment

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Introduction

A MULTIPLE objective optimization technique is applied to the design of modalized observers. The modalized observer was introduced by Andry et al.[1] and involves the choice of closed-loop observer left eigenvectors so that the effect of a known mismatch of initial conditions between the observer and plant is minimized. Such mismatches occur, for instance, in flight control problems when the aircraft (in a straight and level flight condition) is subject to a gust disturbance that introduces nonzero values to the variables angle of attack α and/or sideslip angle β.[1] To minimize the effect of such disturbances, Andry et al.[1] proposed a technique that involves the direct assignment[2] of the left eigenvectors of the observer to match a given prespecified set of vectors that results in a reduced state estimation error dynamic. However, as has been pointed out in Ref. 3, such a process is likely to produce an observer that possesses a set of eigenvalues that are highly sensitive to parameter variations or uncertainties that may occur in the system matrices.

A new approach was proposed[3] that involves the optimization of a multicriterion cost function that takes account of both eigenvalue sensitivity and of estimator error caused by an initial condition mismatch. This multiple objective approach involves the selection of a fixed set of closed-loop observer eigenvalues. The corresponding assignable eigenvector subspaces[2] are then calculated. This freedom for eigenvector assignment is used to reduce the value of the cost function using a quasi-Newton search with numerically evaluated gradients.

The present paper offers a slightly different approach that introduces several improvements over the work by Sobel and Banda.[3] An analysis, similar to that contained in Ref. 3, results in the definition of a new multiobjective cost function J for which analytically derived gradients are available. This constitutes the first improvement since the convergence properties and accuracy of the solution using analytical gradients

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